

# Flavor-Changing at Large $\tan \beta$

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## References:

K.S. Babu and C. Kolda, *Phys. Rev. Lett.* **84**, 228 (2000)

K.S. Babu and C. Kolda, hep-ph/0206310, submitted to *PRL*

C. Kolda and J. Lennon, hep-ph/0209xxx

## Related Talks:

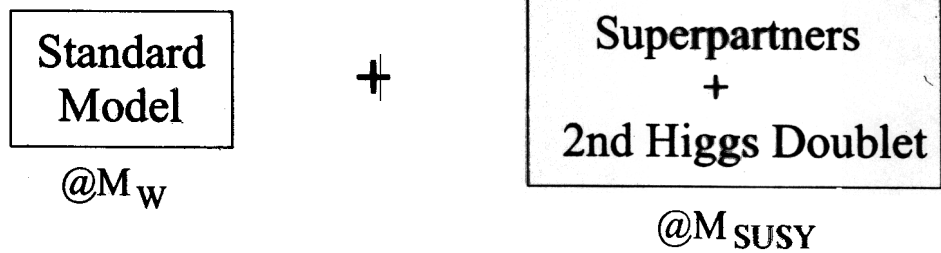
Nierste, Ko

## Conclusions:

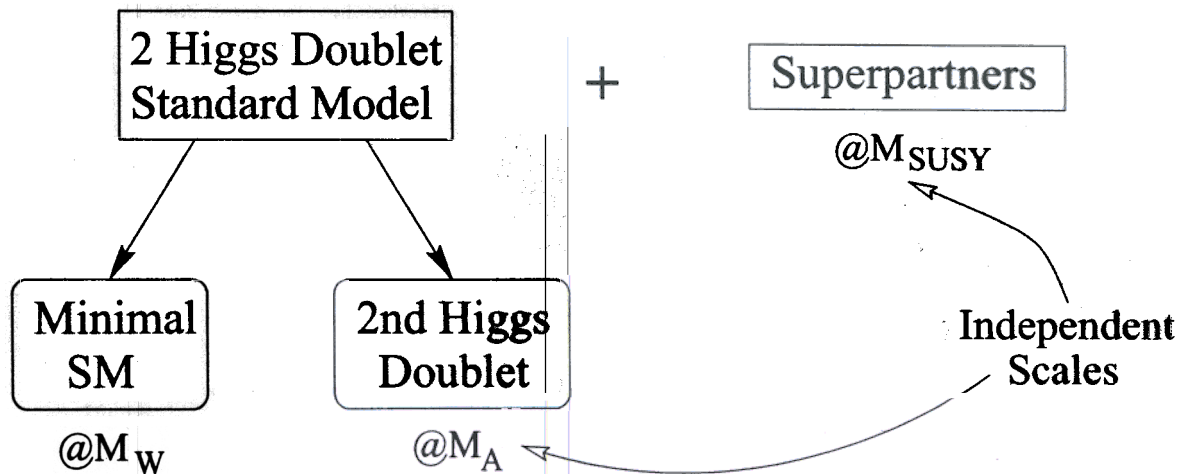
- Within the MSSM, certain classes of FCNC's (e.g.  $B \rightarrow \mu\mu$ ,  $\tau \rightarrow 3\mu$ ) can be mediated by neutral Higgs boson exchange.
- These FCNC's can be large and may be detected before the LHC turns on, and even if SUSY partners are unseen; BR's scale as  $\tan^6 \beta$ !
- These FCNC's decouple differently than all other SUSY-induced FCNC's and are not appreciably constrained by meson-anti-meson mixing amplitudes.
- These FCNC's may provide important clues about method of communicating SUSY-breaking even before we see a single superpartner!

# Philosophy :

We usually think of the MSSM as:



But in some limits it is really better to think like:



Why are there Higgs-induced FCNC's in the MSSM?

*The MSSM is a type-II two Higgs doublet model.*

Separate Higgs doublets give masses to each type ( $u, d$ ) of quark so that Higgs couplings are always  $\propto m_q$ . **→ NO FCNC!**

$$W = QY_u UH_u + QY_d DH_d + \dots$$

GLASHOW,  
WEINBERG

*The MSSM is not a type-II model.*

Type-II models are protected from dangerous  $QuH_d^*$  and  $QdH_u^*$  couplings by a parity:  $H_u \rightarrow H_u$  while  $H_d \rightarrow -H_d$ .

**BUT...**  $W = \dots + \mu H_u H_d$

In MSSM, parity broken by  $\mu$ -terms.

- *The MSSM is a type-II model.*

SUSY doesn't need a parity to protect against dangerous couplings – it has holomorphy!

- *The MSSM really isn't a type-II model.*

Once SUSY is broken, holomorphy fails. Without a parity, nothing to protect against dangerous couplings.

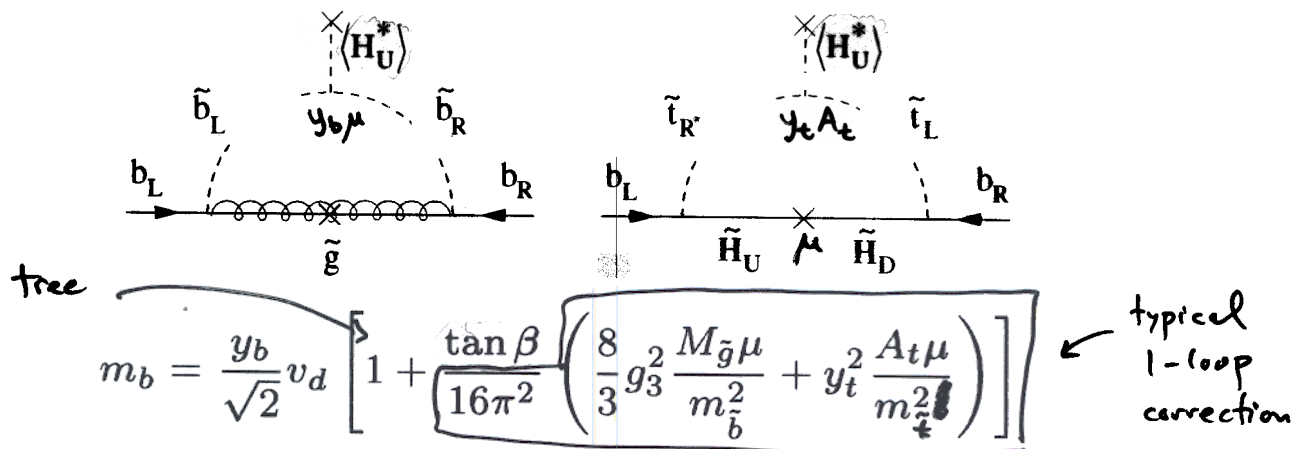
Clearly, dangerous operators must scale as  $\sim \underline{\underline{\mu M_{\text{SUSY}}}}$ .

OLD NEWS : cf. footnote in "Higgs Hunter's Guide"

## SLIGHTLY MORE DETAILS . .

Do the dangerous couplings get generated in real models?

In 1994, Hall, Rattazzi and Sarid examined weak scale corrections to Yukawa coupling unification. At large  $\tan \beta$ , biggest corrections to  $y_{d,s,b}$  come from the  $QdH_u^*$  operator, generated by:



The image shows two Feynman diagrams representing 1-loop corrections to the bottom quark mass. The left diagram shows a bottom quark line (b\_L to b\_R) with a gluon loop (g-tilde) and a Higgs insertion (H\_U\*) with a Yukawa coupling (y\_b mu). The right diagram shows a bottom quark line (b\_L to b\_R) with a Higgs loop (H\_U, H\_D) and a top quark loop (t-tilde\_L, t-tilde\_R) with a Yukawa coupling (y\_t A\_t). Below the diagrams is the formula for the bottom quark mass:

$$m_b = \frac{y_b}{\sqrt{2}} v_d \left[ 1 + \frac{\tan \beta}{16\pi^2} \left( \frac{8}{3} g_3^2 \frac{M_{\tilde{g}} \mu}{m_{\tilde{b}}^2} + y_t^2 \frac{A_t \mu}{m_{\tilde{t}}^2} \right) \right]$$

A handwritten note "typical 1-loop correction" with an arrow points to the loop correction term in the formula. The word "tree" is written next to the first term of the formula.

Even though the effect is formally 1-loop, LARGE  $\tan \beta$  CAN  
OFFSET LOOP SUPPRESSION!

(Similar diagrams exist for  $\delta m_t$  but they are suppressed by  $1/\tan \beta$ .)

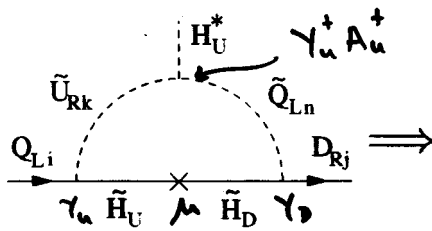
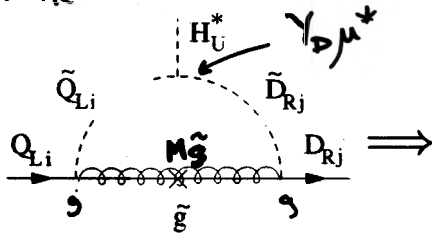
Shortly after, Blazek, Raby and Pokorski put in full flavor structure and showed that large corrections to CKM matrix could be generated.

# HIGGS-MEDIATED FLAVOR-CHANGING NEUTRAL CURRENTS

Begin with the effective Lagrangian in interaction eigenbasis: DIAGONALIZE  
UP SECTOR  
FIRST

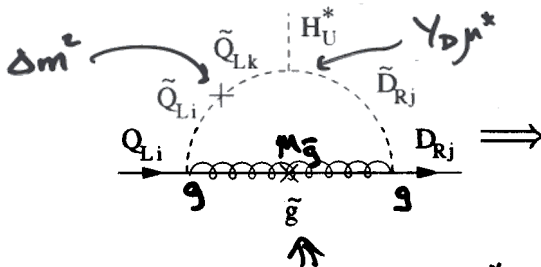
$$-\mathcal{L}_{eff} = \underbrace{\bar{D}_R Y_D Q_L H_d}_{\text{"Tree"}} + \bar{D}_R Y_D \left[ \epsilon_1 + \epsilon_2 Y_U^\dagger Y_U \right] Q_L H_u^* + h.c.$$

$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_R$



$$\epsilon_1 \simeq \frac{2\alpha_3}{3\pi} \frac{\mu^* M_{\tilde{g}}}{m_0^2}$$

$$\epsilon_2 \simeq \left| \frac{1}{\gamma} \frac{\mu^* A_U}{m_0^2} \right| \quad [1]$$



"2-LOOP EFFECT"

$$\frac{d}{dt} m_{\tilde{Q}}^2 \simeq \frac{1}{8\pi^2} (3m_0^2 + A_0^2) [Y_U^\dagger Y_U + Y_D^\dagger Y_D]$$

$$m_{\tilde{Q}}^2 \simeq \bar{m}^2 (1 + c Y_U^\dagger Y_U + c Y_D^\dagger Y_D)$$

$$\Delta \epsilon_2 \simeq -\frac{1}{3} c \epsilon_1 \quad [2]$$

- For universal-ish SUSY-breaking masses,  $\epsilon_2(\tilde{C}) \simeq \pm \epsilon_g/4$ .
- $\epsilon_1$  and  $\epsilon_2(\tilde{C})$  generated even for completely universal soft masses. **(UNLIKE  $K-\bar{K}$ , etc.)**
- However non-universalities required to get  $\epsilon_2(\tilde{g})$  are very generic. For GUT, typically  $-1 \lesssim c \lesssim -\frac{1}{4}$ .
- $\epsilon_2(\tilde{g})$  present even when A-terms are suppressed.

SPLITTING THIRD  
GENERATION FROM  
#1+2.

Keeping only  $y_b$  and  $y_t$  (and skipping lots of boring algebra)

Yukawas and CKM elements are shifted from their tree-level values:

$$m_b \simeq y_b v_d \left[ 1 + (\epsilon_1 + \epsilon_2 y_t^2) \tan \beta \right] \quad \text{HALL, RATTAZZI, SPIRID}$$

$$V_{ub} \simeq V_{ub}^0 \frac{1 + \epsilon_1 \tan \beta}{1 + (\epsilon_1 + \epsilon_2 y_t^2) \tan \beta} \quad \text{BLAZEK, RABY, POKORSKI}$$

➡ For  $\epsilon_2 = 0$ , no change in the CKM elements, corresponding to no new flavor-changing.

➡ But Yukawas/quark masses still shifted by non-zero  $\epsilon_1$ .

Effective FCNC Lagrangian:

$$\begin{aligned} \mathcal{L}_{FCNC} = & \frac{\bar{m}_b V_{tb}^*}{\sqrt{2} v_d \sin \beta} \chi_{FC} \left[ V_{td} \bar{b}_R d_L + V_{ts} \bar{b}_R s_L \right] \\ & \times \left( \cos(\beta - \alpha) h^0 - \sin(\beta - \alpha) H^0 + i A^0 \right) + h.c. \end{aligned}$$

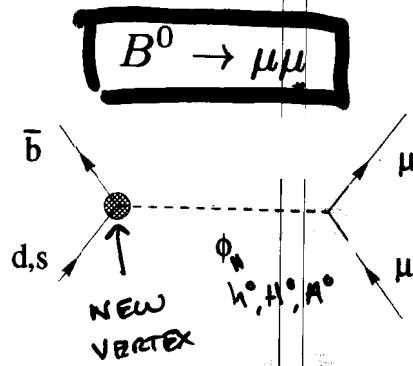
where

$$\chi_{FC} = \frac{-\epsilon_2 y_t^2 \tan \beta}{(1 + \epsilon_1 \tan \beta) \left[ 1 + (\epsilon_1 + \epsilon_2 y_t^2) \tan \beta \right]}$$

and all quarks are in mass eigenbasis.

- Check #1: as  $\epsilon_2 \rightarrow 0$ ,  $\mathcal{L}_{FCNC} \rightarrow 0$ .
- Check #2: as  $m_A \rightarrow \infty$ , contribution of  $h^0$  goes to zero. ( $\alpha \rightarrow \beta - \frac{\pi}{2}$ )

We have generated a  $b_R s_L \phi$  coupling. Where can it appear experimentally?



BABU, CK

Effective Hamiltonian:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} V_{td'}^* V_{tb} [C_{10} \mathcal{O}_{10} + C_{Q1} Q_1 + C_{Q2} Q_2] + h.c$$

with

$$\begin{aligned} \mathcal{O}_{10} &= \frac{\alpha}{\pi} \bar{d}'_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \gamma_5 \ell \\ Q_1 &= -\frac{\alpha}{\pi} \bar{d}'_L b_R \bar{\ell} \ell \\ Q_2 &= -\frac{\alpha}{\pi} \bar{d}'_L b_R \bar{\ell} \gamma_5 \ell \end{aligned} \quad \left. \vphantom{\begin{aligned} \mathcal{O}_{10} \\ Q_1 \\ Q_2 \end{aligned}} \right\} \begin{array}{l} \text{P-S OPERATIONS,} \\ \text{NO HELICITY} \\ \text{SUPPRESSION} \end{array}$$

where  $C_{10}$  is Standard Model and (LARGE  $\tan\beta$ , LARGE  $m_A$ )

$$C_{Q1} \simeq C_{Q2} \simeq \frac{2\pi}{\alpha} \frac{m_b m_\ell}{m_A^2} \chi_{FC}^* \tan^2 \beta \quad \propto \tan^3 \beta$$

$\searrow \propto \tan \beta$

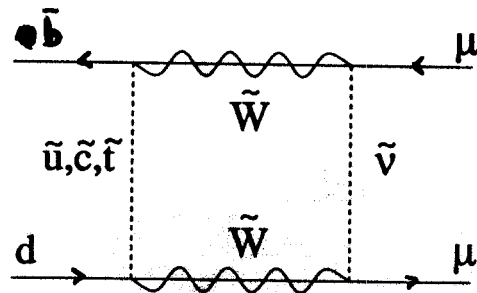
in large  $\tan \beta$  limit.

Then

$$\begin{aligned} \text{BR}(B_{d'} \rightarrow \ell \ell) &= \frac{G_F^2 \alpha^2 m_{B'_d}^3 \tau_{B'_d} f_{B'_d}^2}{64\pi^3} |V_{tb}^* V_{td'}|^2 \sqrt{1 - \frac{4m_\ell^2}{m_{B'_d}^2}} \\ &\times \left[ \left( 1 - \frac{4m_\ell^2}{m_{B'_d}^2} \right) \left| \frac{m_{B'_d}}{m_b + m_{d'}} C_{Q1} \right|^2 + \left| \frac{2m_\ell}{m_{B'_d}} C_{10} - \frac{m_{B'_d}}{m_b + m_{d'}} C_{Q2} \right|^2 \right] \end{aligned}$$

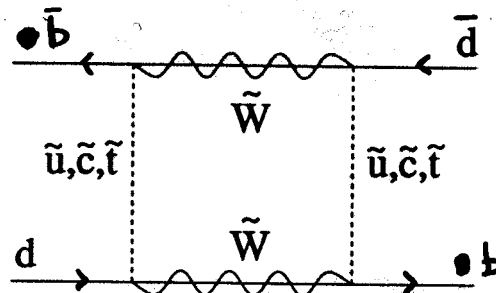
$$\text{BR} \propto \frac{\tan^6 \beta}{m_A^4}$$

Absence of  $B^0 - \bar{B}^0$  mixing potentially important. Most SUSY FCNC decays come from boxes:



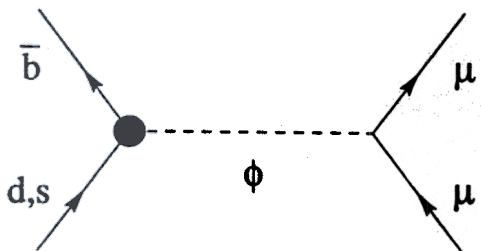
$$\sim \sum_{i=u,c,t} \tilde{V}_{ib}^\dagger \tilde{V}_{id} f(m_i^2)$$

But then there is also mixing:



$$\sim \left[ \sum_{i=u,c,t} \tilde{V}_{ib}^\dagger \tilde{V}_{id} f(m_i^2) \right]^2$$

But here



$$\sim \chi_{FC}$$

While



$$= 0 + \text{HIGHER ORDER}$$

Thus absence of mixing does not imply absence of other FCNC signals!

Things you should know about  $B \rightarrow \mu\mu$ :

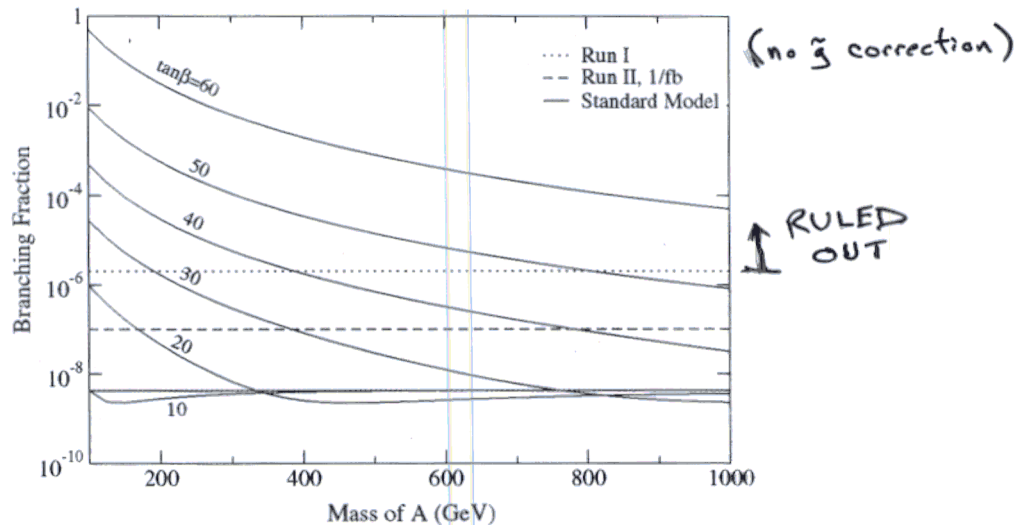
- In the Standard Model,  $B(B_{d,s} \rightarrow \mu\mu) = 1.6 \times 10^{-10}$  and  $4.3 \times 10^{-9}$  (via GIM- and helicity-suppressed penguin)
  - Experimentally,  $Br(B_{(d,s)}^0 \rightarrow \mu\mu) < (6.8, 20) \times 10^{-7}$  at 90% CL (CDF)
  - Relative factor of 3 from relative  $\sigma$  at Tevatron for  $B_d : B_s$ .
  - But theory predicts  $\Gamma_s/\Gamma_d = (V_{ts}/V_{td})^2 \simeq 25$ , so signal in  $B_s$  first.
  - With  $2 \text{ fb}^{-1}$  of data in Run II, a bound of  $(\frac{3}{2} - 1) \times 10^{-7}$  can be obtained. Perhaps another order of magnitude when going to  $15(30) \text{ fb}^{-1}$ . (See ARNOWITT ET AL)
- $\Rightarrow$  Lots of room for SUSY to be found at BR's above SM prediction! **}]!**

(COMPARE TO  $B_d \rightarrow X_s \ell^+ \ell^-$  WHICH GETS ONLY O(1) CORRECTION)

## GENERAL RESULTS

Simplest case: all SUSY masses degenerate

$$M_{\tilde{g}} = \mu = A_t^* = m_{\tilde{q}} = \text{ANYTHING}$$



What are requirements on model for large  $B \rightarrow \mu\mu$ ?

- Large  $\tan \beta$
- Small(ish)  $m_A$
- Large  $\mu$
- Gauginos NOT much lighter than squarks

AND at least one of:

- Large  $A_t(m_Z)$
- Large  $\tilde{b}_L - \tilde{d}_L$  splitting/mixing

★  $B \rightarrow \mu\mu$  does NOT decouple as  $M_{\text{SUSY}} \rightarrow \infty$ , but as  $m_A \rightarrow \infty$ . This is unlike other rare processes ( $b \rightarrow s\gamma$  or  $(g-2)_\mu$ , for example). Thus there can never be perfect correlations between  $B \rightarrow \mu\mu$  and other observables. However correlations can be found in specific models, such as the CMSSM.

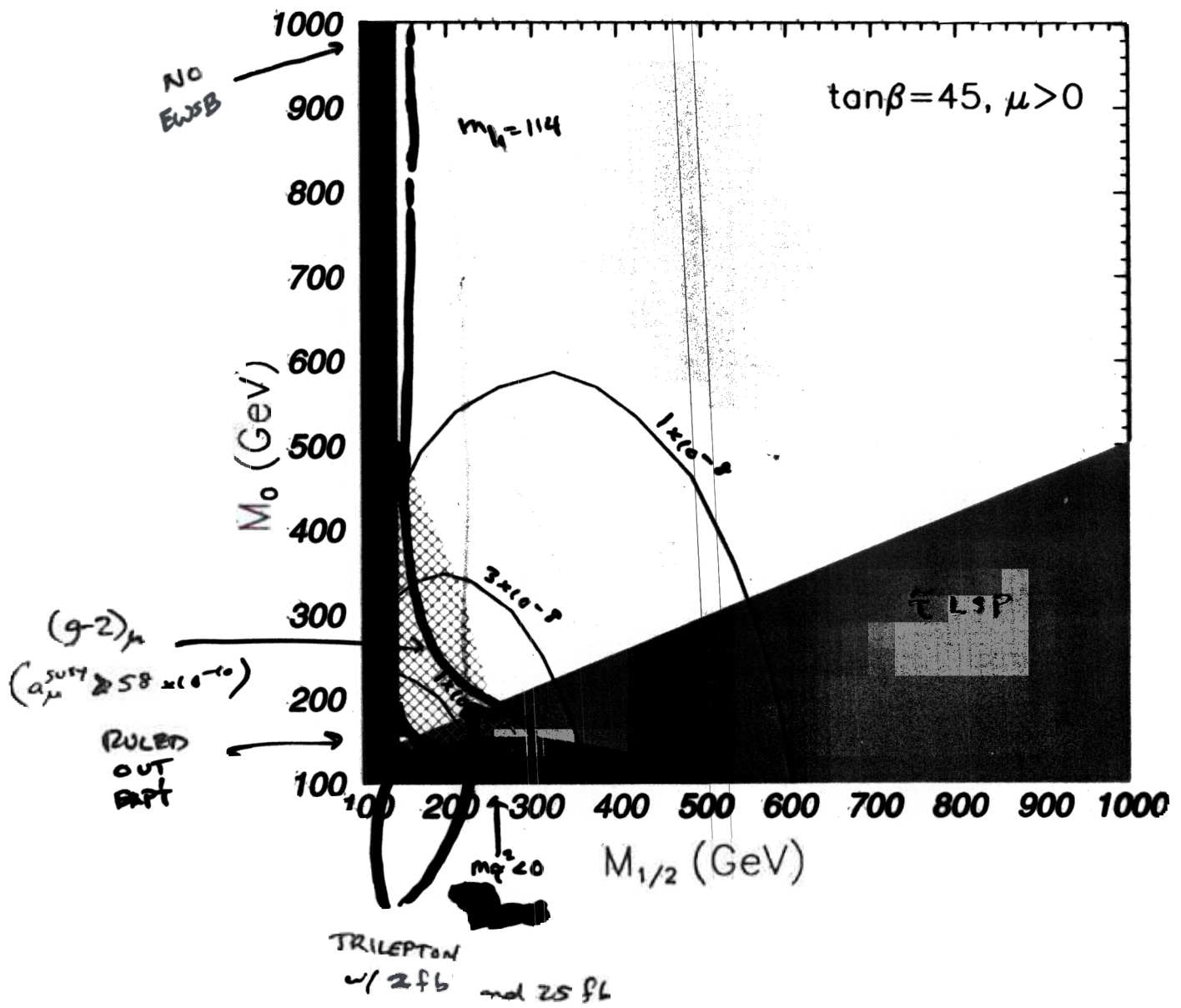
↳ See talks on Saturday!

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$$B \rightarrow \mu\mu : \frac{1}{m_A^2} \bar{b}_R s_L \bar{\mu} \mu$$

$$b \rightarrow s\gamma : \frac{m_b}{m_{\tilde{g}}^2} \bar{b}_R \sigma_{\mu\nu} s_L F^{\mu\nu}$$

$$(g-2)_\mu : \frac{m_\mu}{m_{\tilde{g}}^2} \bar{\mu} \sigma_{\mu\nu} \mu F^{\mu\nu}$$



Some sample models:

### mSUGRA/CMSSM:

- Lots of running typically generates large  $A_t(m_Z)$  and large squark splittings. (3)
- Defining  $M_3 > 0$ , then  $\text{sign}(\epsilon_1)$  is  $\text{sign}(\mu)$ .
- IR pseudo-fixed point of  $A_t$  drives it negative, so  $\text{sign}(\epsilon_2(\tilde{C}))$  is  $-\text{sign}(\mu)$ .
- Third generation squarks split to be lighter than first two generations. Thus  $\text{sign}(\epsilon_2(\tilde{g}))$  is  $\text{sign}(\mu)$ . Thus gluino contribution usually interferes with chargino contribution.
- •  $B \rightarrow \mu\mu$  maximized for  $\mu < 0$  because of cancellations in denominator of  $\chi_{FC}$ .
- • But at large  $\tan\beta$ , mSUGRA models greatly prefer  $\mu > 0$  to avoid large negative contributions to  $(g-2)_\mu$ .
- • Still, for  $\mu > 0$  and  $\tan\beta \gtrsim 25$ , range of Tevatron for finding  $B \rightarrow \mu\mu$  larger than for finding trileptons.

SEE ALSO:

- DEDES, DREINER, NIERSTE  $ph/0108037$
- HUANG, LIAO  $ph/0201121$
- ARNOWITT, DUTTA, KAMON, TANAKA  $ph/0203069$

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BAER, KO, SONG  $ph/0205259$

BAER, BALAZS, BELYAEV, MIZUKOSHI, TATA YANG  $ph/0205325$

## GMSB:

- Predicts  $A \simeq 0$  at messenger scale  $M$  ☹️
- Predicts  $m_{\tilde{d}} = m_{\tilde{s}} = m_{\tilde{b}}$  also at  $M$  ☹️
- If  $M$  is low, then running has no chance to generate  $A$ -terms or squark splittings. ☹️

Generic GMSB models DO NOT predict much of a  $B \rightarrow \mu\mu$  signal beyond the Standard Model.

- Conditions for a signal:
  1. Large messenger scale  $M$  to generate lots of running helps, since running generates  $A$ -terms and mass splittings.
  2. Large  $N$  (# of messengers) helps a little by increasing  $M_{\text{gaugino}}$  w.r.t.  $M_{\text{scalar}}$ .

Baek *et al* find that if Run II sees  $B \rightarrow \mu\mu$ , GMSB with  $N = 1$  and  $M \lesssim 10^{10}$  GeV is ruled out, and any GMSB model with  $\tan\beta \lesssim 50$  is ruled out. ~~any GMSB model with  $\tan\beta \lesssim 50$  is ruled out.~~

## AMSB:

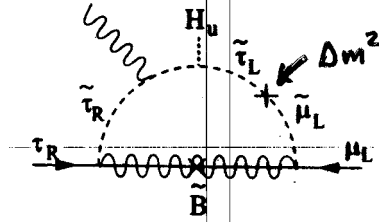
- Baek *et al* find AMSB models also ruled out by  $B \rightarrow \mu\mu$  observation in Run II.
- Our calculation for AMSB not done, but early results contradict this. Difference is probably in details of the  $b \rightarrow s\gamma$  calculation which acts as important constraint in AMSB models.

Flavor-Changing Neutral Currents in the quark sector are “morally equivalent” to charged Lepton Flavor Violation (LFV) in the lepton sector.

- We know  $\nu$ FV exists ( $\nu$ -oscillations) but in SM this shows up in charged leptons suppressed by  $(m_\nu/M_W)^n$ . *Way small!*
- In SUSY, charged slepton flavor violation easier to arrange: can be encoded in non-diagonal slepton masses. If  $\tilde{\text{LFV}}$  is  $O(1)$ , LFV is only suppressed by  $(m_\ell/m_{\tilde{\ell}})^n$ .
- But lack of large FCNC's in quarks *probably* implies mass universalities that *probably* apply to sleptons too.
- Let's assume that  $\nu$ 's get mass through a seesaw with a heavy  $\nu_R$  ( $M_R \sim 10^{14-15}$  GeV) and at least one  $y_\nu \sim O(1)$
- Mass non-universality in squarks sneaks back in through RGE's and the large  $y_t$ .
- Mass non-universality in sleptons sneaks back in through RGE's and the large  $y_\nu$ . BUT only at  $Q^2 > M_R^2$ !
- Well-known in  $\tau \rightarrow \mu\gamma$ . Does it generate Higgs-mediated LFV?

gluino

Reminder of  $\tau \rightarrow \mu \gamma$ : (just like ~~gluino~~ piece in  $b \rightarrow s \gamma$ )



HISANO + many others

Where does the  $\Delta m_{\tilde{\ell}}^2$  come from?

$$\begin{aligned} \rightarrow \frac{d}{d \log Q} (m_{\tilde{\ell}}^2)_{ij} &= \left( \frac{d}{d \log Q} (m_{\tilde{\ell}}^2)_{ij} \right)_{\text{MSSM}} \quad \leftarrow \text{TRIVIAL FLAVOR STRUCTURE} \\ &+ \frac{1}{16\pi^2} \left[ m_{\tilde{\ell}}^2 Y_{\nu}^{\dagger} Y_{\nu} + Y_{\nu}^{\dagger} Y_{\nu} m_{\tilde{\ell}}^2 \right. \\ &\quad \left. + 2(Y_{\nu}^{\dagger} m_{\tilde{\nu}_R}^2 Y_{\nu} + m_{H_u}^2 Y_{\nu}^{\dagger} Y_{\nu} + A_{\nu}^{\dagger} A_{\nu}) \right]_{ij} \quad \leftarrow \text{Non-TRIVIAL} \end{aligned}$$

So, mass insertion is:

$$(\Delta m_{\tilde{\ell}}^2)_{ij} \simeq -\frac{\log(M/M_R)}{16\pi^2} \left( 6m_0^2 (Y_{\nu}^{\dagger} Y_{\nu})_{ij} + 2(A_{\nu}^{\dagger} A_{\nu})_{ij} \right) \equiv \xi (Y_{\nu}^{\dagger} Y_{\nu})_{ij} \quad \leftarrow \text{TAKE } A \propto Y$$

where

$$\xi = -\frac{\log(M/M_R)}{16\pi^2} (6 + 2a^2) m_0^2.$$

What is  $M$ ?

Worst case:  $M_{\text{GUT}}$ .

Best case:  $M_{\text{Pl}} \Rightarrow \log(M/M_R) \simeq 10$ .

$$\text{Br}(\ell_i \rightarrow \ell_j \gamma) \propto (Y_{\nu}^{\dagger} Y_{\nu})_{ij}$$

What do we know about  $Y_\nu$ ??

With large mixing in 2-3 and 1-2, “most popular” ansatz for mass is

$$m_\nu \propto \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}$$

- If  $M_R \propto 1$ , then  $m_\nu \propto Y_\nu^\dagger Y_\nu$
- If  $M_R \simeq 10^{14}$  GeV, then  $(Y_\nu)_{33} \simeq 1$
- In many GUTs, predict  $(Y_\nu)_{33} \simeq y_t \sim 1$

Another option: *inverted hierarchy ansatz*

$$m_\nu \propto \begin{pmatrix} \epsilon & 1 & 1 \\ 1 & \epsilon & \epsilon \\ 1 & \epsilon & \epsilon \end{pmatrix}$$

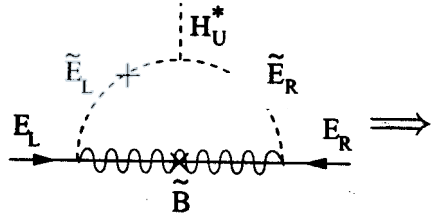
(More on this later...)

## HIGGS-MEDIATED LEPTON FLAVOR CHANGING

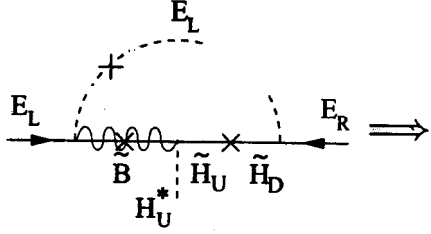
Write an effective Lagrangian: (Just like  $B \rightarrow \mu \mu$ )

$$\mathcal{L} = \bar{E}_R Y_E E_L H_d^0 + \bar{E}_R Y_E \left( \epsilon_1 1 + \epsilon_2 Y_\nu^\dagger Y_\nu \right) E_L H_u^{0*} + h.c.$$

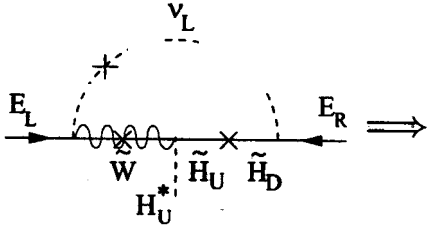
At low scale, no explicit  $Y_\nu$  can appear since  $M_R \gg M_{\text{SUSY}}$ , but can appear as log-enhanced  $\Delta m_{\tilde{\ell}}$  mass insertion.



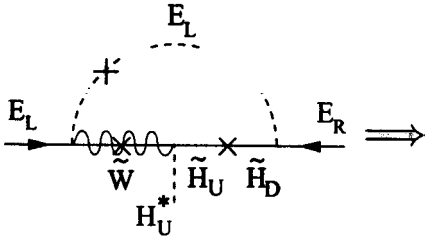
$$\epsilon_2 \simeq \frac{\alpha'}{4\pi} \xi \mu M_1 f_2(M_1^2, m_{\tilde{\ell}_L}^2, m_{\tilde{\tau}_L}^2, m_{\tilde{\ell}_R}^2)$$



$$\epsilon_2 \simeq + \frac{\alpha'}{8\pi} \xi \mu M_1 f_2(M_1^2, m_{\tilde{\ell}_L}^2, m_{\tilde{\tau}_L}^2, \mu^2)$$



$$\epsilon_2 \simeq \frac{\alpha_2}{4\pi} \xi \mu M_2 f_2(M_2^2, m_{\tilde{\nu}_\ell}^2, m_{\tilde{\nu}_\tau}^2, \mu^2)$$



$$\epsilon_2 \simeq \frac{\alpha_2}{8\pi} \xi \mu M_2 f_2(M_2^2, m_{\tilde{\ell}_L}^2, m_{\tilde{\tau}_L}^2, \mu^2)$$

where

$$f_2(a, a, a, a) = \frac{1}{6a^2}, \quad f_2(a, b, b, b)|_{b \ll a} \simeq \frac{1}{2ab}$$

## FLAVOR-CHANGING TAU DECAYS

Some algebra takes us to effective Lagrangian for LFV Higgs couplings:

$$-\mathcal{L} \simeq (2G_F^2)^{1/4} \frac{m_\tau \kappa_{32}}{\cos^2 \beta} (\bar{\tau}_R \mu_L) \left[ \cos(\beta - \alpha) h^0 - \sin(\beta - \alpha) H^0 - i A^0 \right]$$

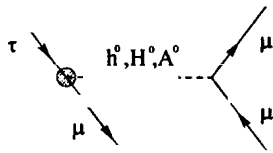
where

$$\kappa_{ij} = - \frac{\epsilon_2}{\left[ 1 + (\epsilon_1 + \epsilon_2 (Y_\nu^\dagger Y_\nu)_{33}) \tan \beta \right]^2} (Y_\nu^\dagger Y_\nu)_{ij}$$

← 1-loop quantity  
(2 loops w/ a large log)

(Lagrangian for  $(\bar{\tau}_R e_L)$ -Higgs derived by  $\kappa_{32} \rightarrow \kappa_{31}$ )

Then  $\tau \rightarrow 3\mu$ :



$\Rightarrow$

$$\text{Br}(\tau \rightarrow 3\mu) = \frac{G_F^2 m_\mu^2 m_\tau^7 \tau_\tau}{768 \pi^3 m_A^4} \kappa_{32}^2 \tan^6 \beta$$

(in large  $m_A$  limit where  $\alpha \rightarrow \beta - \pi/2$ ).

For  $\mu = M_1 = M_2 = m_{\tilde{\ell}} = m_{\tilde{\nu}}$ ,  $M_R = 10^{14}$  GeV and  $(Y_\nu^\dagger Y_\nu)_{32} = 1$ : then

$$\Rightarrow \epsilon_2 \simeq 4 \times 10^{-4}$$

and

$$\text{Br}(\tau \rightarrow 3\mu) = (1 \times 10^{-7}) \times \left( \frac{\tan \beta}{m_A} \right)^6 \times (100 \text{ GeV})^4$$

Since B-factories are also  $\tau$ -factories, BaBar and Belle should be probing the applicable range over the next couple years. LHC and SuperKEKB will have more than  $10^9$   $\tau$ 's.

$$Y_\nu^\dagger Y_\nu \propto \begin{pmatrix} \epsilon & & \\ & \epsilon & \\ & & \epsilon \end{pmatrix}$$

For inverted hierarchy ansatz,  $\tau \rightarrow 3\mu$  is tiny but now  $\tau \rightarrow e\mu\mu$  can be large thanks to large  $(Y_\nu^\dagger Y_\nu)_{13}$ .

Can also observe  $\mu \rightarrow 3e$  (despite *tiny* electron Yukawa!):

$$\text{Br}(\mu \rightarrow 3e) = (5 \times 10^{-14}) \times \left(\frac{\tan \beta}{60}\right)^6 \times \left(\frac{100 \text{ GeV}}{m_A}\right)^4 \times (Y_\nu^\dagger Y_\nu)_{21}^2$$

But from  $\mu \rightarrow e\gamma$  already known that (roughly)

$$(Y_\nu^\dagger Y_\nu)_{21} \lesssim 10^{-2} \times \left(\frac{m_{\tilde{\mu}}}{100 \text{ GeV}}\right)^2 \quad \leftarrow \text{makes it harder!}$$

But if observed, may be **ONLY** way to reconstruct electron Yukawa coupling.

$\tau \rightarrow 3\mu$  and  $\mu \rightarrow 3e$  can also occur with Higgs mediation — take photon off-shell in  $\tau \rightarrow \mu\gamma$  or  $\mu \rightarrow e\gamma$ :

$$\Rightarrow \frac{\text{Br}(\tau \rightarrow 3\mu)}{\text{Br}(\tau \rightarrow \mu\gamma)} \simeq 0.003, \quad \frac{\text{Br}(\mu \rightarrow 3e)}{\text{Br}(\mu \rightarrow e\gamma)} \simeq 0.006$$

Any significant deviation from these ratios would be sign of new physics beyond canonical SUSY sources  $\Rightarrow$  **Higgs mediation!**

Lessons from  $\tau \rightarrow 3\mu$  and related rare LFVs:

Lots of information about  $\nu$ -Yukawa and  $\nu_R$  Majorana mass matrices encoded into BR's. May be hard to decipher but many models could be ruled out with even a single observed rare decay.

SUSY masses entering calculation are generally simple to measure directly (slepton & gaugino masses,  $\mu$ -term,  $\tan\beta$ ) so calculation can be compared easily and  $\xi \propto Y_\nu^\dagger Y_\nu$  extracted.

★ One should expect a (model-dependent) correlation between these processes and  $(g-2)_\mu$  and perhaps  $b \rightarrow s\gamma$ . And of course  $\tau \rightarrow \mu\gamma$  and  $\mu \rightarrow e\gamma$ .

Like  $B \rightarrow \mu\mu$ , observation would probably rule out low-scale gauge-mediation (or low-scale mediation of any sort). Requires high-scale mediation but otherwise has little apparent dependence on the type of model (mSUGRA vs. AMSB, for example)

★★ These are (unique?) windows on the Yukawa coupling of the light leptons and even the neutrinos.